

Temperature dependent weak field Hall resistance in 2D carrier systems

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Using the Drude-Boltzmann semiclassical transport theory, we calculate the weak-field Hall resistance of a two-dimensional system at low densities and temperatures, assuming carrier scattering by screened random charged impurity centers. The temperature dependent 2D Hall coefficient shows striking non-monotonicity in strongly screened systems, and in particular, we qualitatively explain the recent puzzling experimental observation of a decreasing Hall resistance with increasing temperature in a dilute 2D hole system. We predict that the impurity scattering limited Hall coefficient will eventually increase with temperature at higher temperatures.

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The behavior and the properties of the apparent two dimensional (2D) “metallic” phase continue to attract substantial attention [1] from experimentalists and theorists alike, even a decade after its original discovery [2]. In particular, the original observations on the strong metallic (i.e. $d\rho/dT > 0$) temperature dependence of the 2D resistivity, $\rho(T)$, where the resistivity may increase by as much as a factor of 3 – 4 for a modest increase in temperature (e.g. $T = 100mK - 3K$) were followed by intriguing observations of large magnetoresistance in an applied parallel magnetic field. Phenomenologically the observed “metallicity”, defined as the maximum temperature induced enhancement of $\rho(T)$, exhibits strong system dependence, with 2D p-GaAs hole system being the most metallic and 2D n-GaAs electron system being the least metallic with the 2D Si-based electron systems having intermediate metallicity. This system-dependent variation can be understood on the basis of our theoretical prediction [3] that the metallicity arises from 2D screening properties, and is therefore controlled in the zeroth order theoretical prescription by the dimensionless parameters $q_{TF}/2k_F$ and T/T_F , where q_{TF} , k_F , T_F are respectively 2D Thomas-Fermi screening wave vector, the Fermi wave vector, and the Fermi temperature [4]. Since $q_{TF}/2k_F \propto mn^{-1/2}$ and $T/T_F \propto mn^{-1}$ in 2D, the metallicity increases with increasing (decreasing) carrier effective mass m (carrier density n). This is why metallicity is stronger at lower carrier densities and/or in higher effective mass semiconductor systems.

The idea [5] of the strongly temperature dependent screened charged impurity effective disorder being the qualitative reason underlying the striking metallic behavior of dilute 2D carrier systems has come to be known as the “screening theory” since the screening-induced regularization of the bare Coulombic impurity disorder (in contrast to zero-range white noise disorder) intrinsic to semiconductor systems is the crucial physical mechanism in this theory. The screening theory has been applied, with reasonable qualitative success, to explain the temperature and density [3,5,6] dependence as well the parallel magnetic field dependence [7] of the 2D “metallic”

resistivity [3,5]. Motivated by a puzzling recent experimental observation [8], we develop in this Letter the first theory, based on the screening model, for the temperature dependence of the weak-field Hall resistance in dilute 2D carrier systems. Our theory is in excellent qualitative agreement with the experimental results [8] on the temperature dependence of the weak field Hall resistance although quantitative discrepancies remain.

The recent Hall resistance measurements [8] take on particular significance since the experimental data reported by Gao *et al.* [8] disagree *qualitatively* with the so-called “interaction theory” [9,10], which has recently been much discussed and debated in the literature [9–15,3]. The interaction theory complements the screening theory by carrying out a perturbative diagrammatic calculation to include *all* (i.e. not just screening) interaction corrections to the 2D conductivity in the weak-disorder (the so-called “ballistic” regime), low-temperature limit defined by $(\hbar = k_B = 1) \tau_0^{-1} \ll T \ll T_F$, where τ_0 is the $T = 0$ transport (“Drude”) relaxation time, $\sigma_0 \equiv \sigma(T = 0) = ne^2\tau_0/m$. The interaction theory, which purportedly improves and extends the screening theory by including higher-order interaction corrections, has several limitations: (1) the theory is restricted to only small temperature induced corrections $\delta\sigma(T)$ to σ_0 with $\sigma(T) = \sigma_0 + \delta\sigma(T)$, and $|\delta\sigma| \ll \sigma_0$ – as such this theory is, by construction, incapable of explaining the large temperature-induced changes in the conductivity observed in the 2D “metallic” phase; (2) the theory is necessarily restricted to very low temperatures $T \ll T_F$, which may not be achieved experimentally; (3) the theory has only been developed for 2D systems with white-noise bare impurity disorder, i.e. for an unrealistic model of zero-range bare impurity scattering potential (in reality, the impurities in 2D semiconductor structures are random Coulombic charge centers, not zero-range neutral scatterers); (4) the interaction theory predicts only the leading-order temperature dependence (“linear-in T”) for 2D conductivity as $\delta\sigma(T) = \sigma_0 C_1 (T/T_F)$, where the temperature-independent coefficient C_1 is a universal (but unknown) function of density defined by the Fermi

liquid “triplet” interaction parameter F_0^σ . Depending on the value of F_0^σ , $\delta\sigma(T)$ could be positive or negative. In spite of the fact that the only real prediction of the interaction theory for $\sigma(T)$ is that the leading-order thermal correction to the 2D conductivity is linear in T (with an unknown positive or negative coefficient), and the experimental $\sigma(T)$ is rarely linear over any appreciable temperature range, there have been experimental attempts [12–15] to attribute the 2D metallicity to interaction effects by fitting the experimental $\delta\sigma(T) \equiv \sigma(T) - \sigma_0$, where σ_0 is obtained by a linear extrapolation to $T = 0$, to the formula $\delta\sigma(T)/\sigma_0 = C_1(T/T_F)$ and thereby obtain the fitted coefficient C_1 and consequently extract the Fermi liquid parameter F_0^σ . It may be appropriate here to mention that the screening theory also predicts a leading-order linear temperature correction to $\sigma(T)$. While both the screening theory and the interaction theory predict $\delta\sigma(T) \sim T/T_F$, the screening theory is not limited to just the leading-order temperature dependence and can be applied [3,5,6] to calculate the complete $\sigma(T)$ – in fact, the screening theory becomes more accurate at higher temperatures. The screening theory is, however, a self-consistent field theory which only includes the effects of screened effective disorder arising from the charged impurity scattering leaving out all higher-order (i.e. beyond screening) effects of interaction. Therefore, the applicability of the screening theory at very low-density strongly interacting 2D system is suspect and is only of qualitative validity. The importance of the interaction theory [9,10], in spite of its limitations, arises from its very general and universal nature where the temperature correction to the 2D conductivity is linked to universal functions of Fermi liquid interaction parameters.

Since the temperature-dependent conductivity itself (at least, the leading-order temperature correction) cannot distinguish between the interaction and the screening theory (with both predictions being $\delta\sigma/\sigma_0 \propto T/T_F$), it becomes imperative to look for other more definitive signatures for interaction effects in low-density and low-temperature 2D transport properties. This is where the temperature dependence of weak-field 2D Hall resistivity $\rho_H(T)$ takes on great significance as was emphasized in ref. [10], and recently, in ref. [8]. The interaction theory makes [10] very specific (and falsifiable) predictions about the connection between the (leading-order) temperature corrections to 2D conductivity $\delta\sigma(T)$ and Hall resistivity $\delta\rho_H(T)$ where $\rho_H(T) = \rho_H^0 + \delta\rho_H(T)$ with $\rho_H^0 \equiv \rho_H(T \rightarrow 0)$. The predicted connection between $\delta\sigma(T)$ and $\delta\rho_H(T)$, which has been discussed in detail with exemplary clarity in refs. [8] and [10], arises from the fact that in the interaction theory both of these temperature corrections are controlled by the same Fermi liquid parameters, and as such, a detailed and careful measurement of $\sigma(T)$ at low temperatures necessarily *completely* determines the low-temperature behavior of $\rho_H(T)$ in the same sample. Gao *et al.* carried out [8] such a com-

parison between $\rho_H(T)$ and $\sigma(T)$ using the interaction theory [10] in an extremely high-quality 2D p-GaAs hole system which is metallic at extraordinary low densities down to 10^{10} cm^{-2} (where the 2D hole system should be very strongly interacting, making it a perfect system for testing the internal consistency of the interaction theory). The outcome as detailed in ref. [8] is a spectacular *qualitative* failure of the interaction theory: In the density range studied [8] by Gao *et al.* the Fermi liquid parameters extracted from their experimental $\sigma(T)$ data should lead to, according to the interaction theory results of ref. [10], essentially a constant temperature-independent $\rho_H(T)$ except at the lowest temperature the interaction theory predicts a small (less than 1%) increase in $\rho_H(T)$ with increasing temperature, whereas Gao *et al.* find a smooth *decrease* ($\sim 20\%$) in $\rho_H(T)$ with increasing temperature as T varies from 100 mK to 1K. Thus the interaction theory disagrees with the temperature dependent Hall resistance data of ref. [8], both qualitatively and quantitatively.

In this Letter we present the first calculated results for the 2D temperature dependent Hall resistance using the screening theory formalism. We solve the semiclassical Drude-Boltzmann transport equation [16] for the weak-field Hall resistance finding that the 2D Hall resistivity ρ_H can be written as ($c = 1$)

$$\rho_H = \frac{B}{ne} r_H, \quad (1)$$

where B is the applied weak magnetic field and r_H , the so-called Hall ratio [4], is given by

$$r_H = \langle \tau^2 \rangle / \langle \tau \rangle^2, \quad (2)$$

where τ is the (energy and temperature dependent) 2D carrier transport scattering time (the so-called momentum relaxation time) determined by the *screened* charged impurity scattering [3], and $\langle \tau \rangle$ is the thermal average over the finite temperature Fermi distribution function – the detailed definitions of τ and $\langle \tau \rangle$ are given in the Appendix of ref. [3] and will not be repeated here. We note that $\langle \tau \rangle$ determines the Drude-Boltzmann conductivity through the relation $\sigma(T) = ne^2 \langle \tau \rangle / m$.

At $T = 0$, $\langle \tau^2 \rangle \equiv \langle \tau \rangle^2$, giving $r_H = 1$ so that $\rho_H(T = 0) \equiv \rho_H^0 = B/ne$, the classical Hall formula. At finite temperatures it is well known that the Hall ratio $r_H \neq 1$ due to thermal corrections, and below we present our results for the calculated temperature-dependent Hall resistance in the Drude-Boltzmann theory of screened charged impurity scattering. Following the standard notation we define the Hall coefficient $R_H = d\rho_H/dB = r_H/ne$.

Our numerical results (shown below) for the temperature dependent Hall resistance are calculated assuming only carrier scattering by screened charged impurity scattering, leaving out phonon scattering (except in the inset of Fig. 2(a)). For screening the Coulombic disorder

potential from the unintentional random charged impurities in the background and interfaces we use the random phase approximation (RPA) augmented by the Hubbard approximation (HA) for the local field correction [17,18]. It is well-known that HA quantitatively improves upon the RPA, particularly at the very low carrier densities of interest [8], by including some short-range exchange-correlation effects beyond the self-consistent field approximation of RPA. Our calculations are done for the realistic GaAs/Ga_{1-x}Al_xAs quantum well systems used in ref. 9 with the finite-width form-factor effects [3,4] associated with the quantum well confinement included in the theory. The charged impurities are assumed to be uniformly randomly distributed in the background and interfaces, which set the overall resistivity scale (i.e. σ_0^{-1}) in the system. All calculations are done for a quantum well width of 100Å corresponding to the sample of ref. [8].

In Fig. 1 we show our calculated conductivity $\sigma(T)$ and Hall coefficient $R_H(T)$ (inset) as a function of temperature for several hole densities. Both $\sigma(T)$ and $R_H(T)$, in agreement with the experimental data [8], decrease with increasing temperature, exhibiting some interesting non-monotonicity that we discuss below. The overall decrease for $\sigma(T)$, about a factor of 3 – 4, and for $R_H(T)$, about 10 – 20%, are in excellent agreement with experiment although there are quantitative discrepancies in the details, which is to be expected given the highly approximate nature of our theory.

The most significant quantitative discrepancy between experiment and theory is that the overall temperature scale for the theory is somewhat wider than that in the experiment [8]. While this problem can be somewhat rectified by including additional scattering mechanisms (e.g. phonons, surface roughness, alloy disorder, remote impurities) which are invariably present in real 2D semiconductor systems, our goal in this paper is to avoid excessive data fitting (which would not be particularly meaningful from the perspective of fundamental understanding of the 2D metallic phase) in order to establish a basic zeroth order qualitative understanding of the temperature dependent 2D Hall effect. We therefore accept the quantitative discrepancy as the signature of the approximate nature of our theory, emphasizing the fact that the theory seems to be an excellent qualitative description of the experimental observations in ref. [8].

In Fig. 2 we show our calculated Hall ratio $r_H(T)$ and the Hall coefficient, $R_H(T)$, as a function of temperature. In particular, for $R_H(T)$ we carry out a direct comparison between our screening theory and the experimental results from ref. [8] (cf. Fig. 2(b) in ref. [8]). For the sake of comparison the interaction theory result [10] for $R_H(T)$, adopted from Fig. 2(b) of ref. [8], is also shown on the same plot. First, we note that while the interaction theory disagrees qualitatively and radically with the experimental results, our screening theory is in very good qualitative agreement. The screening theory catches well

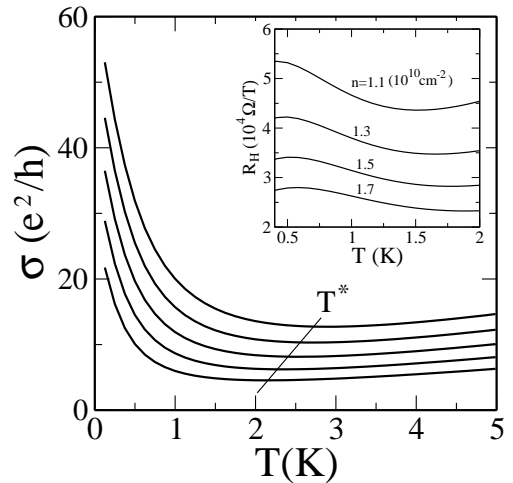


FIG. 1. Calculated impurity scattering limited conductivity $\sigma(T)$ as a function of temperature for different densities $n = 1.1, 1.3, 1.5, 1.7, 1.9 \times 10^{10} \text{ cm}^{-2}$ (bottom to top). T^* indicates the temperature at which $d\sigma/dT$ changes sign. Inset shows the calculated Hall coefficient $R_H(T)$ for different densities.

the overall magnitude ($\sim 20\%$) of the temperature induced *decrease* in the Hall coefficient although there are quantitative discrepancies. The most important discrepancy is that the temperature scale for the temperature dependence is off by about 400 mK in the sense that the experimental temperature scale (the bottom abscissa in Fig. 2(b)) goes from 0 to 1.5K whereas our theoretical scale (the top abscissa of Fig. 2(b)) goes from 0.5K to 2.0K. This same discrepancy can be seen in Fig. 1 also where T^* , the temperature scale where $d\sigma/dT$ changes sign at finite temperatures (the so-called “quantum-classical crossover” [5] phenomenon), is consistently higher by roughly a factor 3 in the theory (compare, for example, T^* in our Fig. 1(a) with the corresponding experimental T^* in Fig. 1 of ref. [8]). Currently, we have no good explanation for this discrepancy in the actual value of T^* — it could, for example, be arising from an enhanced effective mass (e.g. due to many-body electron-electron interaction) or due to the effects of other scattering mechanisms such as remote impurities, which is known to suppress the theoretical T^* . For our qualitative theory of the temperature dependent Hall effect we just accept this discrepancy in the temperature scale and shift our theoretical results by 0.5 K in Fig. 2(b), getting excellent qualitative agreement between experiment [8] and the screening theory.

In discussing the overall temperature dependence of the Hall ratio r_H shown in Fig. 2 (a), where $R_H \equiv r_H/(ne)$, we see that there is a very striking non-monotonicity in r_H over the temperature scale 0 – 4K which has not been observed experimentally. But, the experimental results of ref. [8] clearly suggest tantalizing signs of possible non-monotonic behavior both at the low

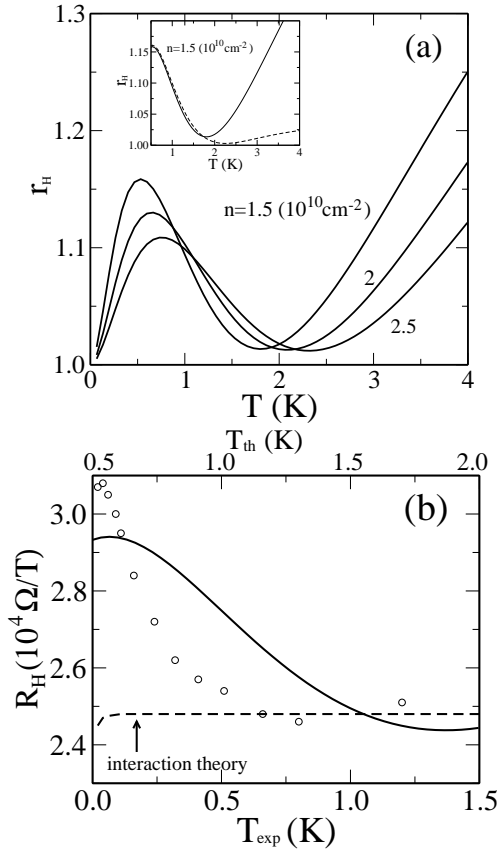


FIG. 2. (a) Calculated Hall ratio $r_H(T)$ as a function of temperature for different densities by considering only screened charged impurities. The phonon contribution (dashed line) is given in the inset for $n = 1.5 \times 10^{10} \text{ cm}^{-2}$. (b) Calculated Hall coefficient with experimental data [8] and the result of the interaction theory for hole density $n = 1.65 \times 10^{10} \text{ cm}^{-2}$. The scale of R_H is set by experiment [8].

and the high temperature ends with the data in ref. [8] unfortunately ending (both at low and high temperatures) precisely where the non-monotonicity may just be appearing. Since the temperature scale of our screening theory is off by about 500 mK, the non-monotonicity at the low temperature end (i.e. the initial rise of r_H with increasing T) is probably not observable because carrier heating is likely to prevent real low carrier temperatures from being achieved. The higher temperature rise of $r_H(T)$, beyond $T > 2\text{K}$ in Fig. 2(a), is a true prediction of our theory which should be experimentally tested, but this eventual increase of $r_H(T)$ with T is likely to be masked and considerably suppressed by phonon scattering effects which become qualitatively important in low-density p-GaAs 2D hole systems of interest here already at $T \geq 1\text{K}$ (see the inset of Fig. 2(a)). Our preliminary estimate of phonon scattering effects on the temperature dependent Hall resistivity (the results shown in Figs. 1 and 2, we emphasize, include only carrier scattering by background screened charged impurities) suggests that

the maximum value of r_H is unlikely to exceed 1.1, and therefore the size of the non-monotonicity is suppressed as shown in the inset of Fig. 2(a).

Before concluding, we point out that, according to our screening theory, the decreasing $R_H(T)$ with increasing temperature observed in Ref. [8] and manifest in our Figs. 1 and 2 is obviously *not* an asymptotic leading-order temperature behavior, and as such, it is missed entirely in the interaction theory which predicts a qualitatively different behavior. We can calculate the analytic leading order temperature dependence of the Hall ratio $r_H(T)$ in the screening theory and find it to be $r_H(T) = 1 + \frac{\pi^2}{3}(\frac{T}{T_F})^2$, i.e. $\delta\rho_H(T) \sim T^2$, in the strictly 2D limit. This is different from the leading-order prediction of the interaction theory, but this is understandable since the origin of the temperature correction in R_H is different in the two theories: in the screening theory it is a Fermi surface averaging effect at finite temperatures whereas in the interaction theory, it is a true many-body Fermi liquid renormalization effect. Our results, as compared with experimental data of ref. [8], demonstrate that the asymptotic leading-order theoretical temperature dependence is not meaningful in understanding the temperature dependence of the 2D Hall coefficient — the temperature dependence of $R_H(T)$ arises from non-asymptotic higher-order temperature effects and the asymptotic leading-order temperature dependence remains inaccessible experimentally because it happens at extremely low temperatures. We have argued elsewhere [3] that the same may be true for the temperature dependence of the 2D longitudinal resistivity also, which is rarely linear in the experimental data.

We have developed the screening theory for the temperature dependent Hall effect in 2D “metallic” systems. Our results are in reasonable qualitative agreement with the recent experimental observations of Gao *et al.* [8]. We explain why the experimental data [8] disagree qualitatively with the interaction theory [10] by suggesting that the asymptotic leading-order temperature dependence of the interaction theory is simply inaccessible experimentally and is therefore physically uninteresting. We predict that the Hall coefficient, which decreases [8] with increasing temperature upto the highest measurement temperature ($\sim 1.2\text{K}$), will eventually increase by 5 – 10% when the temperature is increased to 2K or beyond, showing an interesting non-monotonic behavior.

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